

Homework 07

Name: _____

Wes Email: _____

Question	Points	Score
1	25	
Total:	25	

If possible, please type/write your answers on this sheet and upload a copy of the PDF to your google drive handin folder. Otherwise, please write the answers in some sort of word processor and upload a PDF. Please name the file `hw07-written.pdf`.

1. Map Fusion

Earlier in the course, we had a function `raiseBy : int list * int -> int list` that added its `int` argument to each element of the `int list`. We proved that for all values `l : int list, a: int, b: int`,

$$\text{raiseBy}(\text{raiseBy}(l, a), b) \cong \text{raiseBy}(l, a + b)$$

This is a special case of a property called *map fusion*. Recall the `map` function:

```
fun map (f : 'a -> 'b, l : 'a list) : 'b list =
  case l of
  [] => []
  | x :: xs => f x :: map (f, xs)
```

Mapping `f` over some list `l` and then mapping another function `g` over the result gives a list that is equivalent to the one you would get if you map `fn x => g (f x)` (“`g` composed with `f`”) over the original `l`. We can write this more concisely by defining an abbreviation for function composition, the function that applies `f` and then applies `g`:

```
fun (g : 'b -> 'c) o (f : 'a -> 'b) = fn x => g (f x)
```

The property we would like to prove is that for all lists `l`,

$$\text{map } (g \circ f, l) \cong \text{map}(g, \text{map } f \ l)$$

Unfortunately, this is *false* for certain `g` and `f`. We say that a function `f` is *total* if for all values `v`, `f v` is valuable: that is, a function is total iff it is valuable on all inputs.

- (5) (a) Give functions `g` and `f` and a list `l` such that `map (g o f, l` and `map (g, map (f, l))` have different behaviors. Hint: consider `f` and `g` that are not total.

Solution:

However, we *can* prove this property for total f and g .

You may assume the following lemma:

Lemma 1. *For all types a , b and values $f : a \rightarrow b$, if f is total then $\text{map}(f, l)$ is valuable.*

Your job is to prove

Theorem 1. *For all types a , b , c , all values $f : a \rightarrow b$ and $g : b \rightarrow c$, and $l : a$ list if f and g are total, then*

$$\text{map}(g, \text{map}(f, l)) \cong \text{map}(g \circ f, l)$$

Proceed by induction on the structure of l . **Be careful to note where you are using valuability, and explain why the expressions involved are valuable—where would your proof break for the non-total functions in your example above?**

(5) (a) **Solution:** Case for $[]$:

To show:

- (15) (b) **Solution:** Case for $x : xs$, where x and xs are values:
IH:

To show:

Solution: (continue here if necessary)