Lecture 16

- sequences + sequence analysis

- start n-body simulation

$\boxed{\text{Arrays}} \longrightarrow$ persistent

1	2	7	11	...

$\boxed{\underline{\text{'a}}\ \ \text{seq}}$

"sequence"

'a $\underline{\text{Seq}}$.seq

module name

```
type 'a Seq.seq

val Seq.length : 'a Seq.seq -> int
val Seq.nth : int * ('a Seq.seq) -> 'a
val Seq.map : ('a -> 'b) * 'a Seq.seq -> 'b Seq.seq
val Seq.reduce : ('a * 'a -> 'a) * 'a * 'a Seq.seq
        -> 'a
```

Seq.length

Behavior

$$\text{Seq.length}(\ [\ x_0 \mid \cdots \mid \cdots \mid x_{n-1}\ ]\ ) = n$$

Work

Span

$$O(1)$$

$\boxed{\text{Seq.nth}}$

Behavior $\text{Seq.nth}\,(i, \overbrace{\boxed{x_0 \mid x_1 \mid \ldots}}^{s}) = x_i$ if $0 \le i < \text{length}(s)$

or raise Range

Work $O(1)$

Span

Seq.map

Behavior

$$\text{Seq.map}\left(f,\ \boxed{x_0 \mid x_1 \mid x_2 \mid \cdots}\right)$$

$$= \boxed{f(x_0) \mid f(x_1) \mid f(x_2) \mid \cdots}$$

	f is O(1)	otherwise
Work	$O(n)$	sum $W_f(x_0) + W_f(x_1) + \cdots$
Span	$O(1)$	max $S_f(x_0), S_f(x_1), \cdots$

Seq. reduce Behaviour

$$\text{Seq.reduce}(\oplus, b, [x_0 | x_1 | x_2 | \ldots])$$

$$= x_0 \oplus x_1 \oplus x_2 \oplus x_3 \oplus x_4 \ldots \oplus x_{n-1}$$

or $= b$ if length $= 0$

$$\mathrm{reduce}\left(\tilde{+}, \tilde{0}, \boxed{7 \mid 8 \mid 3 \mid 12}\right)$$

$$= (7+8)+(3+12) \quad = ((7+8)+3)+12$$

$$= 7+(8+(3+12))$$

$$\mathrm{reduce}\left(+, 0, \boxed{7.0 \mid 8.9 \mid 3.1}\right)$$

$$= (7.0+8.9)+3.1$$

$$= 7.0+(8.9+3.1)$$

"associativity"

$$(x+y)+z \overset{?}{=} x+(y+z)$$

If ⓝ associative, then "parens" don't matter

$$\text{reduce}(ⓝ, b, \boxed{x_0 | x_1 | \cdots}) = x_0 ⓝ x_1 ⓝ x_2 ⓝ \cdots$$

If ⓝ isn't, picks a parenthesization

→ balanced

not balanced

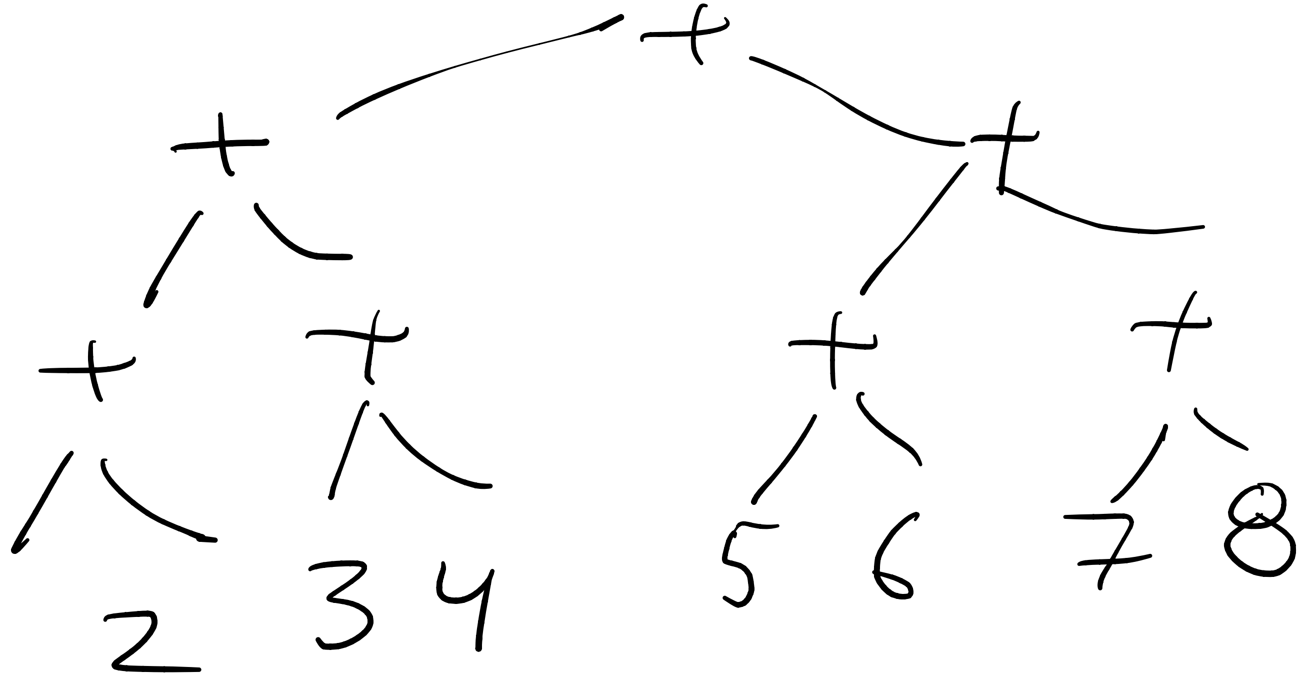
$$x_0 + (x_1 + (x_2 + (x_3 + (x_4 + \cdots))))$$

$$((((x_0 + x_1) + x_2) + x_3) + x_4) + \cdots$$

is balanced

$$((x_0 + x_1) + (x_2 + x_3)) + ((x_4 + x_5) + (x_6 + x_7))$$

work $O(n)$



balanced

span $O(\log n)$

is

reduce(n, b, s)

n is $O(1)$

reduce(n, b, s) where n is not $O(1)$

↳ pretend it's your HW7 code on a balanced tree

use tree method

Problem: Add all #s in a grid of numbers

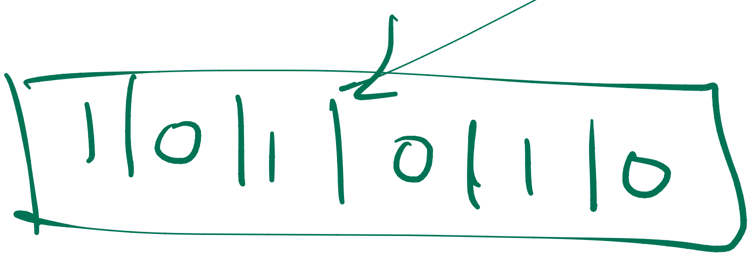
(int Seq.seq) Seq.seq

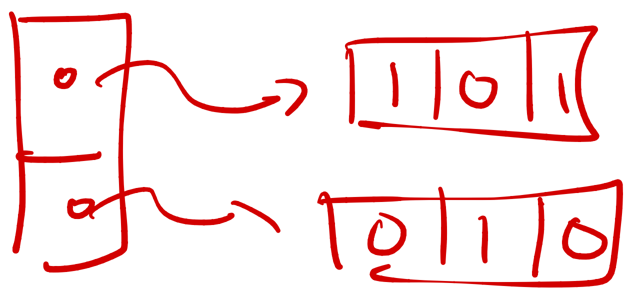
1, 0, 1
0, 1, 0

$\rightsquigarrow$

$\langle \langle 1, 0, 1 \rangle,$
$\langle 0, 1, 0 \rangle \rangle$

angles mean arrays





① fun sum(s: int Seq.seq): int =
Seq.reduce (+, 0, s)

e.g. sum([0 | 1 | 1]) = 2 = 0+1+1

② fun count(c: (int Seq.seq) Seq.seq): int =
sum (Seq.map (sum, c))

outer

int Seq.seq → int

int Seq.seq → col. final

$$\text{count}\left(\langle 1, 0, 1\rangle, \langle 0, 1, 1\rangle\right)$$

$$\mapsto \text{sum}\left(\text{map}\left(\text{sum}, \left(\langle\langle 1, 0, 1\rangle, \langle 0, 1, 0\rangle\rangle\right)\right)\right)$$

$$\mapsto \text{sum}\left(\langle \text{sum}\langle 1, 0, 1\rangle, \text{sum}\langle 0, 1, 0\rangle\rangle\right)$$

$$\mapsto \text{sum}\left(\langle 1+0+1, 0+1+0\rangle\right)$$

$$\mapsto \text{sum}\left(\langle 2, 1\rangle\right) \mapsto 2+1 \mapsto 3$$

	input size	work	span
inner sum	n	$O(n)$	$O(\log n)$
map	$n \times n$ grid	each row $\times n$ $O(n^2)$	$O(\log n)$
outer sum	n	$O(n)$	$O(\log n)$
overall	$n \times n$	$O(n^2)+O(n) = O(n^2)$	$O(\log n)+O(\log n) = O(\log n)$

work $O(n^2)$

span $O(\log n)$

$$\frac{\text{procs}}{\text{can use}} \approx \frac{n^2}{\log n} \quad \frac{W}{S}$$

N-body simulation

n celestial bodies

"planet"

predict positions using
basic (non-relativistic)
physics

1 Steps of motion

S – position of a body

V – velocity

a – acceleration

$$s' = s + vt + \frac{1}{2}at^2$$

where is a body after some time

velocity

accel

$$v' = v + a * t$$

new velocity

old velocity

accel

time

$$a' = ?$$

what is accel?

Newton's 2nd law

Force = mass * accel

$$\text{accel} = \frac{\text{Force}}{\text{mass}}$$ → how much is other stuff pushing/pulling on body

$$a = \frac{F}{m}$$

assume knowledge

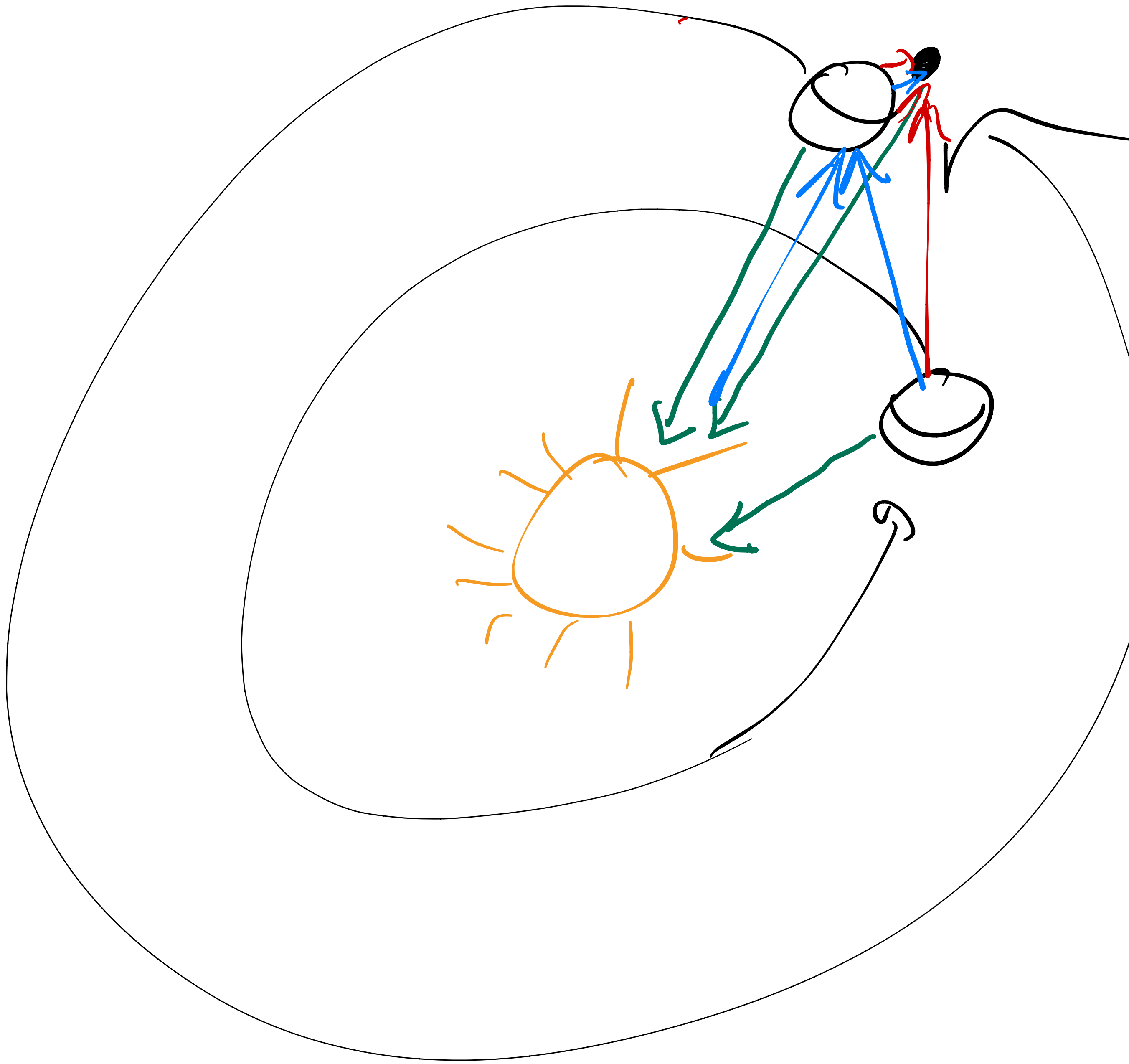
F → force on a body

Newton's law of gravitation

$$F_i = \underset{j \neq i}{\text{sum}}\ \underbrace{F_{ij}}_{\text{force on body } i \text{ due to body } j}$$

force on body i (earth)

$$F_{ij} = G \frac{\overbrace{m_i}^{\text{mass of body } i}\ \overbrace{m_j}^{\text{mass of body } j}}{\underbrace{(r_{ij})^2}_{\text{distance between two}}}$$



$$\underset{\text{accel on body } i}{a_i} = \frac{F_i}{m_i}$$

$$= \frac{\mathrm{sum}_j\, F_{ij}}{m_i}$$

constant

$$= \frac{\mathrm{sum}_j \left(\frac{G\, m_i m_j}{(r_{ij})^2} \right)}{m_i}$$

mass of body j

$$\boxed{a_i = \mathrm{sum}_j \left(\frac{G\, m_j}{(r_{ij})^2} \right)}$$

distance between i and j