COMP 212 : Functional Programming, Fall 2024

Homework 05

Name: _____

Wes Email: _____

Question	Points	Score
1	15	
2	18	
Total:	33	

If possible, please type/write your answers on this sheet and upload a copy of the PDF to your google drive handin folder. Otherwise, please write the answers in some sort of word processor and upload a PDF. Please name the file `hw05-written.pdf`.

See the homework handout for descriptions of the problems.

1. **Proof**

(15)   (a) Prove the following:

Theorem 1. *For all trees* `t`, `depth t ≤ size t`.

Solution: The proof is by structural induction on `t`.

Case for `Empty`

To show:

Proof:

Solution: Case for `Node(l,x,r)`

Inductive hypotheses:

To show:

Proof:

(more space for the proof if you need it)

2. **Rebalance**

(3)   (a) Give a recurrence for the work of `takeanddrop` in terms of the **depth** d; argue that it is $O(d)$.

> **Solution:** $W_{\texttt{takeanddrop}}(d) =$

(3)   (b) Give a recurrence for the span of `takeanddrop` in terms of the **depth** d; argue that it is $O(d)$.

> **Solution:** $S_{\texttt{takeanddrop}}(d) =$

(3)   (c) Give a recurrence for the work of `halves` in terms of the **depth** d; give a tight big-O bound.

Solution: $W_{\texttt{halves}}(d) =$

(3)   (d) Give a recurrence for the span of `halves` in terms of the **depth** d; give a tight big-O bound.

Solution: $S_{\texttt{halves}}(d) =$

(3)   (e) Give a recurrence for the work of `rebalance` in terms of the **size** n; assume the depth of the tree is $c \log n$ for some c. Give a tight big-O bound. Show your work using a closed form or sum.

Solution: $W_{\texttt{rebalance}}(n) =$

(3)   (f) Give a recurrence for the span of `rebalance` in terms of the **size** n; assume the depth of the tree is $c \log n$ for some c. Show your work using a closed form or sum.

Solution: $S_{\texttt{rebalance}}(n) =$