

Lecture 13:

Polymorphism

+

Data types

Poly morphism:

Some code works  
on

multiple types  
of data

# Type constructors:

a way of making types  
from smaller types

A  $\boxed{\text{Int}}$   $\boxed{\text{list}}$  is  
 $\boxed{()}$ , or

$x :: xs$  where  $x : \boxed{\text{Int}}$   
 $xs : \boxed{\text{Int}} \boxed{\text{list}}$

A  $\boxed{\text{Str}}$   $\boxed{\text{list}}$  is  
 $\boxed{()$ }, or

$x :: xs$  where  
 $x : \boxed{\text{Str}}$   
 $xs : \boxed{\text{Str}} \boxed{\text{list}}$

An  $\boxed{\text{Ia}}$  list is  
 $- \boxed{()}$ , or

$- x :: xs$  where

$x : \boxed{\text{Ia}}$   
 $xs : \boxed{\text{Ia}} \boxed{\text{list}}$

bool list

[true, false]

~~list list~~ list

[ [1,2], [3,4] ]

(

<

List<Bool>

fun length (l: int list): int =  
case l of  
| [] => 0

$$| x :: xs \Rightarrow 1 + \text{length } xs$$

length [1, 2]

$\alpha = \text{int}$

fun length (l:  $\alpha$  list): int =  
case l of  
| [] => 0

$$| x :: xs \Rightarrow 1 + \text{length } xs$$

fun length (l: string list): int =  
case l of  
| [] => 0

$$| x :: xs \Rightarrow 1 + \text{length } xs$$

length ("a", "b")

$\alpha = \text{string}$

for any type  $\alpha \dots$

fun sum(l: ~~list~~<sup>'a</sup> list): int =  
case l of

( ) => 0

| x :: xs =>  $x + \underbrace{\text{sum}(xs)}_{\text{int}}$

type error: 'a + int ???

~~~~~

Sum(true, false, true)  
 $'a = \text{bool}$

fun zip(  $l_1$ : <sup>'a</sup>list,  $l_2$ : <sup>'b</sup>list ): (  $\langle \text{int} * \text{str} \rangle$  ) list =  
 case (  $l_1$ ,  $l_2$  ) of  
     ( [], \_ )  $\Rightarrow$  []  
     | ( \_, [] )  $\Rightarrow$  []  
     | ( x :: xs, y :: ys )  $\Rightarrow$  ( x, y ) :: zip( xs, ys )

$\lambda$   
 $\lambda a = \text{int}$        $'b = \text{string}$

for all  $\alpha, \beta, \gamma \dots$

$\alpha$  list  
 $\alpha$

$\alpha$  list  
 $\beta$

$\alpha$  list  
 $\gamma$

fun append ( $l_1 : \cancel{\text{list}}$ ,  $l_2 : \cancel{\text{list}}$ ) :  $\cancel{\text{list}}$  list =

case  $l_1$  of

$\lambda \Rightarrow l_2$

|  $x :: xs \Rightarrow \underline{x :: \text{append}(xs, l_2)}$

$\alpha$  list

$\alpha = \gamma$

$\beta = \gamma$

# Type inference

fun append(l<sub>1</sub>, l<sub>2</sub>) =  
    case l<sub>1</sub> of

    | []  $\Rightarrow$  l<sub>2</sub>  
    | x::xs  $\Rightarrow$  x :: append(xs, l<sub>2</sub>)

$$l_1 = \text{list}$$

$$l_2 = \text{list}$$

$$x = \text{list}$$

# Data types

An int list is

- [], or
- $x :: xs$  where  $x : \text{int}$   
 $xs : \text{int list}$

→ and that's it!

datatype int list =

[]

| so of int \* int list

;;(x, xs)

A tree is

Empty

Node(l, x, r)

where

l: tree

x: int

r: tree

→ and that's it!

datatype tree =

Empty

| Node of tree \* int \* tree

datatype bool =

true

false

(e<sub>1</sub>, e<sub>2</sub>) : T<sub>1</sub> \* T<sub>2</sub>

where

e<sub>1</sub>: T<sub>1</sub>

e<sub>2</sub>: T<sub>2</sub>

Datatype constructor:

◦ Create values 1::[::C])

◦ pattern match

Case e of

$$P_1 \Rightarrow e_1$$

$$\{ P_2 \Rightarrow e_2$$

⋮

$$\} P_n \Rightarrow e_n$$

Patterns  $P$  are

◦ Made from

- variables

- constructors

◦ Match values

① First match

② Exhaustive

non-exhaustive warning

③ Non-redundant

PatternMatch:  
ValueBinds

1

1

nothing

Xanything  
~~anything~~ ✓~~X to ✓  
nothing~~ $(P_1, P_2)$  $(V_1, V_2)$  when  
 $P_1$  matches  $V_1$   
 $P_2$  matches  $V_2$ what  $P_1$   
and  $P_2$  bind

[]

[]

nothing

 $P_1 :: P_2$  $V_1 :: V_2$  $P_1$  and  $P_2$  bind

Empty

Empty

nothing

Node p

Node v

what p does

case 1::2::[] of

$$[] \Rightarrow \underline{0}$$

$$1 [x] \Rightarrow \underline{1}$$

$$2 x :: (y :: ys) \Rightarrow \underline{2}$$

case  $([], \_)$  of

$([], \_) \Rightarrow 1$



$| (-, []) \Rightarrow 2$

A red oval encloses a vertical bar pointing to the number 2. Above the oval, the word "or" is written.

$| (x :: xs, y :: ys) \Rightarrow 3$

case  $\boxed{[]}$  of

$[] \Rightarrow 0$

A red oval contains the text "non-exhaustive".

$) x :: (y :: ys) \Rightarrow 1$

A red rounded rectangle contains the text "Raises Match exception".

A red oval contains the text "not all values match some pattern".

case 0 of

x => 1

| y => 2

| - => 3

→ 1

redundant

Code

(C), ~~0~~ =>

(C), ~~x::x~~ =>

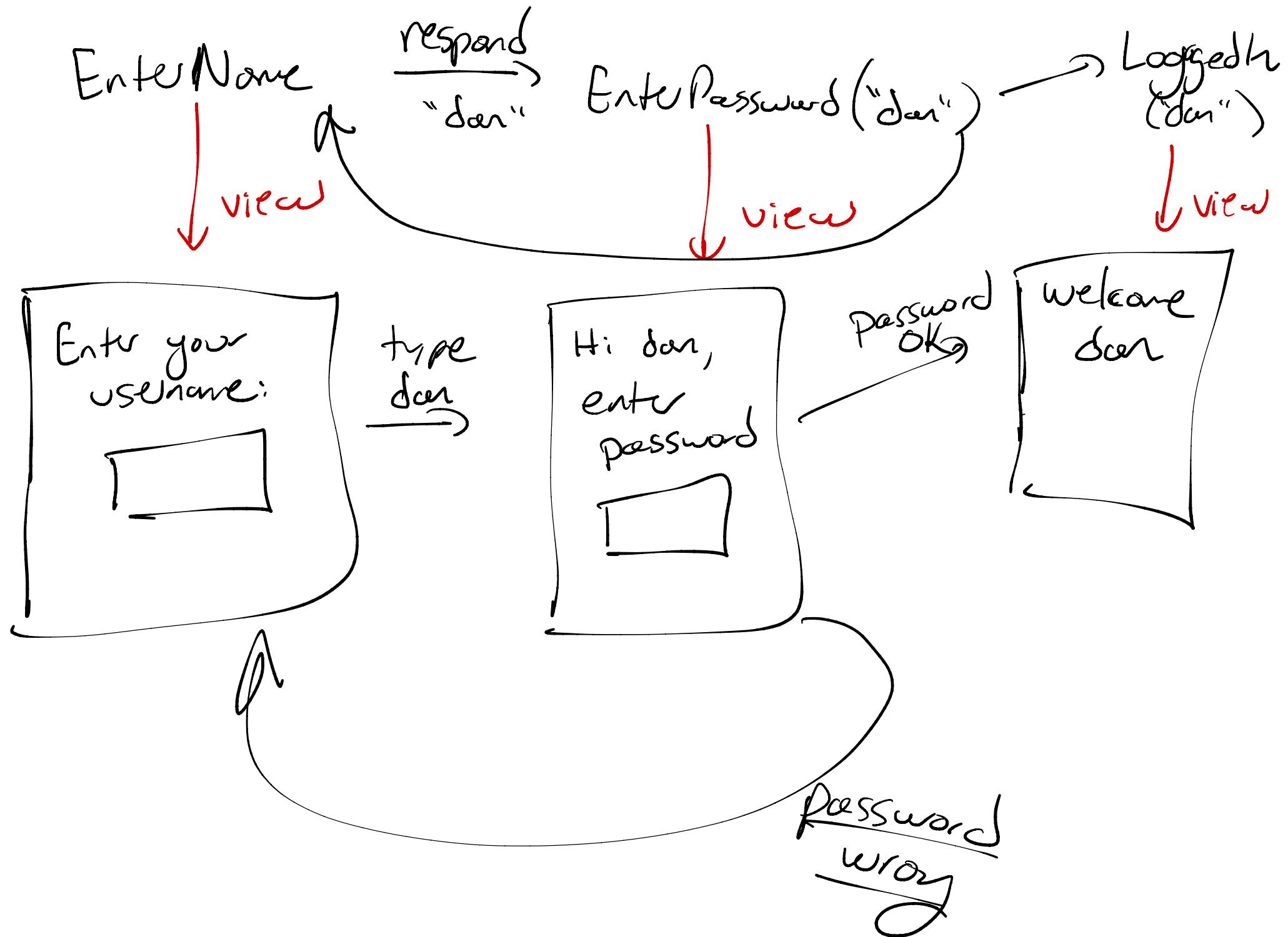
error

Interactive applications:

Model - view - ~~respond~~ - Controller

Separate the logic

from the display/  
front end (s)



datatype model =

EnterName

- | EnterPassword of String (\* name \*)
- | LoggedIn of String (\* name \*)

① view the model as a string

② respond to user input  
by transitioning to a  
new model

(\* input is what the user typed in \*)

fun respond(m: model, input: string): model =

case m of

EnterName  $\Rightarrow$  EnterPassword(input)

| EnterPassword(name)  $\Rightarrow$

(case checkPassword(name, input) of

true  $\Rightarrow$  LoggedIn(name)

| false  $\Rightarrow$  EnterName )

| LoggedIn  $\Rightarrow$  - - -

```
fun view(m: model): string =  
  case m of  
    EnterName => "Please enter your name"  
  | EnterPassword(name) =>  
    "Hi" ^ name ^ "Please enter your password"  
  | LoggedIn(name) =>  
    "Welcome" ^ name ^ " --- "
```

# Controller:

In a loop,

- ① display the view of the model
- ② receive user input and compute the next model based on the respond

H W: shopping cart

- add thgs to cart

- pay

- . . . -

