Lect 21: Machine Learning

Statistical

- mathematical model
- training it on <u>input</u> <u>data</u>
- use it to make predictions

classifier

input → classification

document classification → category

email → spam or not

news article → category
econ / govt / sports

Supervised → have training data with correct outputs

learn to classify test data

Unsupervised → (don't)

Naive Bayes text categorization

Bayes' rule:

number between 0 and 1

$P(A)$ probability of A

event: e.g.

- candy cookie
- no candy cookie
- candy bite
- no candy bite

$P(\underline{\text{no candy bite}}) = \frac{1}{3} = 0.333\ldots$

$P(A)$ probability that A happens

$P(B|A)$ probability that B happens <u>assuming</u> that A happens

<u>conditional probability</u>

$$P(\text{no candy bite} \mid \text{candy cookie}) = \frac{1}{3}$$

$$P(\text{no candy bite} \mid \text{no candy cookie}) = 1$$

Bayes' rule: flip a conditional probability around

$$P(B \mid A) = \frac{P(A \mid B)\, P(B)}{P(A)}$$

$$P(\text{candy cookie} \mid \text{no candy bite}) = \frac{P(\text{no candy bite} \mid \text{candy cookie})\, P(\text{candy cookie})}{P(\text{no candy bite})}$$

$$P(\text{candy cookie} \mid \text{no candy bite}) = \frac{P(\text{no candy bite} \mid \text{candy cookie})\, P(\text{candy cookie})}{P(\text{no candy bite})}$$

$$= \frac{\overbrace{\frac{1}{3}}^{\text{observation}} \times \frac{9}{10}}{\frac{9}{10} \times \frac{1}{3} + \frac{1}{10} \times 1 = \frac{12}{30}}$$

base rate

prior

$$= \frac{\frac{9}{30}}{\frac{12}{30}} = \frac{9}{12} = \boxed{\frac{3}{4}}$$

posterior dist

bite = test

candy bite = negative test

no candy bite = positive test

candy cookie = not infected

no candy cookie = infected

$$P(\text{not infected} \mid \text{positive test}) = \frac{P(\text{positive test} \mid \text{not infected}) \times P(\text{not infected})}{P(\text{positive test})}$$

$P(\text{positive test} \mid \text{not infected}) \longrightarrow$ false positive rate

$P(\text{not infected}) \longrightarrow$ community transmission

$P(\text{positive test}) \longrightarrow$ positive test rate

$\boxed{\text{Middletown}}$

actual = 3x tested

PCR test false positive rate 0.6%

$\times \left(1 - \underbrace{\frac{27}{46{,}5000}}_{\text{not infected}}\right)$

$$= \frac{0.006 \times 0.00058}{0.0262}$$

$$\boxed{= 23\%}$$

positive test rate 2.62% positive

Naive Bayes Text Classification

$$P(\text{spam} \mid \cancel{\text{email}}\ \text{words}) \approx \frac{P(\text{words} \mid \text{spam})\, P(\text{spam})}{\cancel{P(\text{words})}}$$

$$P(\text{not spam} \mid \cancel{\text{email}}\ \text{words}) \approx \frac{P(\text{words} \mid \text{not spam})\, P(\text{not spam})}{\cancel{P(\text{words})}}$$

$$\boxed{\text{training data}}$$

$$\underset{= \text{ base rate}}{P(\text{spam})} = \frac{\#\text{ spam emails}}{\#\text{ emails}}$$

$$P(\underset{w_1, w_2 \ldots w_n}{\text{words}} \mid \text{spam}) \xrightarrow[\text{bag of words}]{\text{naive}} \prod_{i=1}^{n} P(\underset{w_i}{\text{word}} \mid \text{spam})$$

$$= \prod_{i=1}^{n} \frac{\text{count of } w_i \text{ in spam}}{\text{total } \# \text{ of words in spam}}$$

$$P(\text{spam} \mid \underset{w_1 \ldots w_n}{\text{words}})$$

$$= \frac{\#\text{ spam emails}}{\#\text{ emails}} \times \prod_{i=1}^{n} \frac{\text{count of } w_i \text{ in spam}}{\text{total \# words in spam}}$$