

COMP 212 : Functional Programming, Spring 2023

Homework 05

Name: _____

Wes Email: _____

Question	Points	Score
1	15	
2	18	
Total:	33	

If possible, please type/write your answers on this sheet and upload a copy of the PDF to your google drive handin folder. Otherwise, please write the answers in some sort of word processor and upload a PDF. Please name the file `hw05-written.pdf`.

See the homework handout for descriptions of the problems.

1. **Proof**

- (15) (a) Prove the following:

Theorem 1. *For all trees t , $\text{depth } t \leq \text{size } t$.*

Solution: The proof is by structural induction on t .

Case for Empty

To show:

Proof:

Solution: Case for Node(1,x,r)

Inductive hypotheses:

To show:

Proof:

(more space for the proof if you need it)

2. Rebalance

- (3) (a) Give a recurrence for the work of `takeanddrop` in terms of the **depth** d ; argue that it is $O(d)$.

Solution: $W_{\text{takeanddrop}}(d) =$

- (3) (b) Give a recurrence for the span of `takeanddrop` in terms of the **depth** d ; argue that it is $O(d)$.

Solution: $S_{\text{takeanddrop}}(d) =$

- (3) (c) Give a recurrence for the work of **halves** in terms of the **depth** d ; give a tight big-O bound.

Solution: $W_{\text{halves}}(d) =$

- (3) (d) Give a recurrence for the span of **halves** in terms of the **depth** d ; give a tight big-O bound.

Solution: $S_{\text{halves}}(d) =$

- (3) (e) Give a recurrence for the work of **rebalance** in terms of the **size** n ; assume the depth of the tree is $c \log n$ for some c . Give a tight big-O bound. Show your work using a closed form or sum.

Solution: $W_{\text{rebalance}}(n) =$

- (3) (f) Give a recurrence for the span of **rebalance** in terms of the **size** n ; assume the depth of the tree is $c \log n$ for some c . Show your work using a closed form or sum.

Solution: $S_{\text{rebalance}}(n) =$