COMP 212 : Functional Programming, Spring 2025

Homework 05

Name: _____

Wes Email: _____

Question	Points	Score
1	15	
2	18	
3	6	
Total:	39	

If possible, please type/write your answers on this sheet and upload a copy of the PDF to your google drive handin folder. Otherwise, please write the answers in some sort of word processor and upload a PDF. Please name the file `hw05-written.pdf`.

See the homework handout for descriptions of the problems.

1. **Proof**

(15) (a) Prove the following:

Theorem 1. *For all trees* `t`, `depth t` $\leq$ `size t`.

Solution: The proof is by structural induction on `t`.

Case for `Empty`

To show:

Proof:

Solution: Case for `Node(l,x,r)`

Inductive hypotheses:

To show:

Proof:

(more space for the proof if you need it)

2. **Rebalance**

(3) (a) Give a recurrence for the work of `takeanddrop` in terms of the **depth** d; argue that it is $O(d)$.

Solution: $W_{\texttt{takeanddrop}}(d) =$

(3) (b) Give a recurrence for the span of `takeanddrop` in terms of the **depth** d; argue that it is $O(d)$.

Solution: $S_{\texttt{takeanddrop}}(d) =$

(3) (c) Give a recurrence for the work of `halves` in terms of the **depth** d; give a tight big-O bound.

Solution: $W_{\texttt{halves}}(d) =$

(3) (d) Give a recurrence for the span of `halves` in terms of the **depth** d; give a tight big-O bound.

Solution: $S_{\texttt{halves}}(d) =$

(3) (e) Give a recurrence for the work of `rebalance` in terms of the **size** n; assume the depth of the tree is $c \log n$ for some c. Give a tight big-O bound. Show your work using a closed form or sum. You may use the following tight bounds:

$$\log n + \log \tfrac{n}{2} + \log \tfrac{n}{4} + \log \tfrac{n}{8} + \ldots + 1 \quad \text{is} \quad O(\log n)^2)$$
$$\log n + 2 \log \tfrac{n}{2} + 4 \log \tfrac{n}{4} + 8 \log \tfrac{n}{8} + \ldots \text{(with } \log n \text{ terms)} \quad \text{is} \quad O(n)$$

Solution: $W_{\texttt{rebalance}}(n) =$

(3) (f) Give a recurrence for the span of `rebalance` in terms of the **size** n; assume the depth of the tree is $c \log n$ for some c. Show your work using a closed form or sum. You may use the following tight bounds:

$$\log n + \log \tfrac{n}{2} + \log \tfrac{n}{4} + \log \tfrac{n}{8} + \ldots + 1 \quad \text{is} \quad O(\log n)^2)$$
$$\log n + 2 \log \tfrac{n}{2} + 4 \log \tfrac{n}{4} + 8 \log \tfrac{n}{8} + \ldots \text{(with } \log n \text{ terms)} \quad \text{is} \quad O(n)$$

Solution: $S_{\texttt{rebalance}}(n) =$

3. **Cargo**

(3) (a) Analyze the work of your `cargo` function.

Solution:

(3) (b) Analyze the span of your `cargo` function.

Solution: